Explaining Thai and Japanese Student Errors in Solving Equation Problems: The Role of the Textbook

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This paper concerns algebraic errors committed by secondary school Thai and Japanese students. The first author created the Equation Test in the two languages, and the study showed that both groups of students made errors concerning the equalising property and in understanding the verbal problems. However the Thai and Japanese students also committed different types of errors, and the paper relates these errors to the style of problems found in their respective textbooks.

The process of learning algebraic ideas will always generate student errors which, together with their reasons, need to be understood and dealt with by teachers (Booth, 1984; Herscovics, 1989). This study identifies some of the types of errors which students from two countries commonly make in equation problems and proposes some reasons for those errors.

In Thailand, many studies have been focused on the topic of equations, and the results of these studies will also be familiar to most researchers. For instance, Kittiwisit (1994) studied "A Construction of a Mathematics Diagnostic Test" focusing on equations, and found that for the first test, the students' weaknesses concerned the order of division and multiplication properties. For the second test, the students' weaknesses were found to be in mistranslation problems. Similar results and difficulties have been found in other international contexts (Kieran, 1992; Bednarz, Kieran & Lee, 1996).

Explaining these kinds of difficulties is an important task for researchers, and many approaches have been taken. Concerning the study reported here, the first author took the opportunity of being a participant in the 21st In-Service Training Program for Overseas Teachers to observe many classes in Japanese schools. Being from Thailand herself, she became particularly interested in comparing the algebraic skills of secondary school students in Thailand and Japan.

Analysing the textbooks used in the different schools also became an important part of the study. Texts and textbooks are a significant feature of most mathematics classrooms, even in the information technology age (Love & Pimm, 1996). It is clear also that textbooks are for the use of the teachers and students, and are usually specifically written for a particular country and pedagogical culture (Kang & Kilpatrick, 1992). There are wide differences in textbook formats used around the world, and whilst educators are aware of the textbooks' influence on students' learning there is little research comparing the effects of different formats on specific difficulties and errors for students in different countries. This study contributes to the development of that knowledge.

Outline of the Study

The subjects of this study were first grade students in selected lower secondary schools in Khon Kaen, Thailand and in the Saitama prefecture, Japan. The 162 Thai students comprised 70 males and 92 females, and the 136 Japanese students comprised 68 males and 68 females.

The first author created the equation test in the two languages, after consulting many previous research studies. They were checked by back-translation with the help of native speakers and interpreters, and the equation test was open-ended in format. (An English version of the test is shown in the Appendix) The errors committed by the students were classified, tallied, and analysed.

Overall Results

As can be seen the test was composed of 4 items involving solving equations followed by 3 items of verbal problems. The percentage of correct answers is shown in Table 1.

Table 1
Student's problem solving skill

| | Percentage of answer (%) | | | | | |
| | Thai students | | | Japanese students | | |
Item	correct	incorrect	missing	correct	incorrect	missing
1	92.6	5.6	1.8	81.2	14.5	4.4
2	45.1	37.7	17.3	40.3	49.3	10.5
3	45.7	32.1	22.2	89.9	10.1	0
4	45.7	30.9	23.5	89.6	10.5	0
5	34.0	42.6	23.5	56.5	24.6	18.8
6	35.2	28.4	36.4	0	31.3	68.7
7	0	13.6	86.4	19.1	28.7	52.2

Table 1 indicates that the Thai students found most of the items difficult to solve except item number 1 where they got the highest percentage of correct answers. Item number 7 was the most difficult with no-one solving it and 86.4% omitting it. The highest incorrect percentage of any answer was item number 5, 42.6% and the lowest was item number 1, 5.6%.

The Japanese students could solve items 3, 4 as well as 1 with high percentages of correct answers, 89.9%, 89.6 % and 81.2% respectively. Apart from those they were relatively weak on the other items. None of them could solve item number 6. The maximum incorrect percentage was item 2, 49.3% and the minimum was item 3, 10.1%. The maximum missing answer percentage was item 6, 68.7% and item 7, 52.2%.

The table shows further that both Thai and Japanese students could generally answer item 1 (whether correctly or incorrectly), perhaps because it is a simple equation and easy to attempt to solve. Also the Japanese students did not fail to answer items 3 and 4, while almost one quarter of the Thai students did not attempt an answer for either equation.

Analysis of Thai Student Errors

For the first part of the test, the most frequent errors committed were while using the equalising property, that is errors in changing the signs or terms when equalising both sides of an equation. Since the equations were not particularly complicated, this point may imply that the students tried to solve them without first having a good foundation or a mastery of the (previously learned) concepts, in this case the concept of equalising.

For the verbal questions, several errors due to language translation difficulties were also observed. As we can see from Table 1 only about 45% of the Thai students could

solve items 5 and 6, and none could solve item 7. Now let us look at the kinds of errors made on some particular items, with some speculations about the reasons for these errors.

Item (2) 5x – 3/2 = 1

The students have a tendency to solve this equation by using the concept of the least common multiple. Since they learned the least common multiple concept in the first semester they are more familiar with that idea than with the new mathematical concept of equalising. But they did not have a mastery of that concept so they made some errors as shown below:
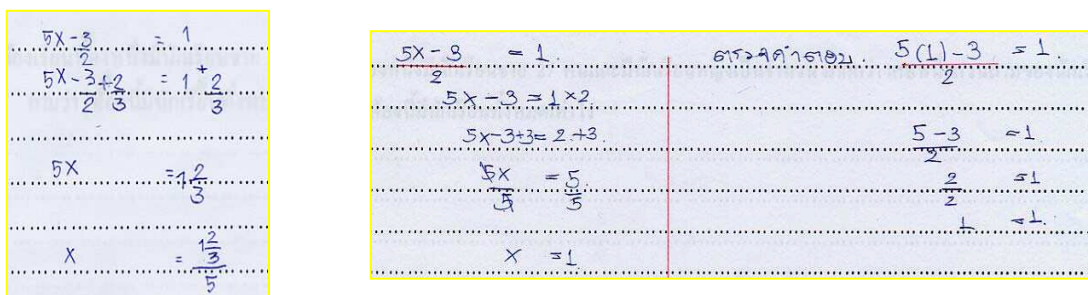
Figure 1. Samples of some errors in Item 2.

Item (3) 6x – 3 = 2x – 19 and item (4) 5x + 3 = 3x – 5,

Items 3 and 4 were both created from the same idea of having each unknown variable on both sides of the equation, and it is interesting that for both these items, 45.7% of the Thai students managed to solve them. The errors shown by the students are due to the weakness of the equalising concept, and in sign changing.

One of the reasons for these errors could be that the mathematics textbook of the Thai schools show most equations as having one unknown variable on one side only. So the students were not familiar with the equations in items 3 and 4. Unfamiliarity and not having much experience to face and solve that kind of unknown variable on both sides of the equation could be the cause of the errors. Samples of the errors are below:
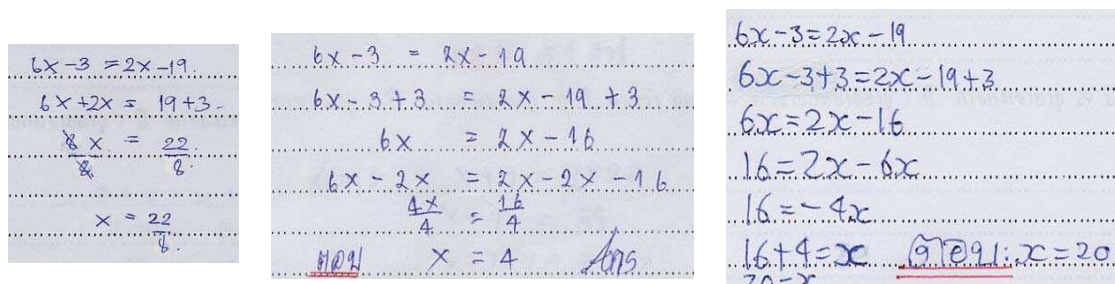

Figure 2. Samples of some errors in Item 3 and 4.

For the verbal problems, as expected, the main errors were with parsing and a failure to understand the problem. Most of the Thai students lacked the ability to correctly translate the problems into equations.

In items 5 and 6, the pattern of the problems is taken from the students' textbook, which could account for the fact that the percentage of correct answer of Thai students in item number 6 is higher than that of the Japanese students. A sample of answers given by the Thai students is below:

Item (5) Sayoo-jan has some money in her wallet. Her father gave her three times more than she already has. She now has 1000 yen. How much money did she have before her father gave her any?

The common error in item number 5 was that students forgot to use all the data about the problem, especially the fact that before her father gave money to Sayoo-jan (3x), she also has her money before (x) so almost all students wrote a wrong equation as shown below:

Figure 3. Sample of an error in Item 5.

Item (6) There are 21 boys in a class and the girls comprise 3 more than 1/3 of all the students. How many girls in a class?

This is another example where the students forgot to use all of data of the problem, especially the fact that the girls comprise 3 *more than* 1/3 of all the students. The difficulties with this question are also shown by the fact that the students tried to use all the numbers included in the problem without understanding the problem, as this example shows:

Figure 4. Sample of an error in Item 6.

Item (7) At present Boy A is 12 years old and Mr. B is 39 years old. When will the age of Mr.B become twice the age of Boy A?

For item 7, the pattern of the problem was the same as those from the textbook of the Japanese schools (MONBUSHO, 1997), which was different from those of the Thai students, and which apparently caused none of the Thai students to be able to solve it.

Analysis of Japanese Student Errors

Japanese students were able to solve simple equation more frequently than the verbal problems. However the percentage of correct answers for items 1, 3 and 4 are higher than that for item 2. Why can only 40% of Japanese students solve item 2?

Based on their answers, it seems that the Japanese students lacked a good understanding of the equalising property especially the division and multiplicative aspects. A sample of their errors is shown below:

1) $5x - 3/2 = 1$
$10x - 3 = 2$
$10x = 2 + 3$
$10x = 5$
$x = 2$

$10x - 6 = 2$
$10x = 2 + 6$
$10x = 8$
$x = 2$

$50x - 15 = 10$
$50x = 10 + 15$
$50x = 25$
$x = 2$

Figure 5. Samples of errors in Item 2.

Also one can see from the students' textbook that there are many examples of equations but most of them involve integers, with few examples involving fractions as shown below. Moreover, from their answers, all the Japanese students used the equalising property to solve item 2, and no-one used the concept of least common multiple to solve the item as the Thai students did. Likewise the textbook contains examples which demonstrated the process of using the equalising property to solve the question as shown below.

例題2 方程式 $9x - 5 = 22$ を解きなさい。

解答▶

$9x - 5 = 22$

両辺に5を加えると、 $\quad 9x - 5 + 5 = 22 + 5$

$9x = 27$

両辺を9でわると、 $\quad x = 3$

答 $x = 3$

Figure 6. An example from a Japanese Textbook (p.77).

Items 5 and 6 adopt the pattern of the problems from the Thai textbook, which could be the reason why the percentage of correct answers of Japanese students is low especially for item 6. Missing this experience meant that the Japanese students made many mistakes such as:

Figure 7. Samples of some errors in Item 6.

For item 7, the pattern of the problem was taken from the Japanese textbook but still only a few Japanese students could solve it, perhaps because it was very difficult for them. However what was particularly interesting was that even if students could not solve problem s 5, 6 or7, they tried to think them out by creating symbols or diagrams.

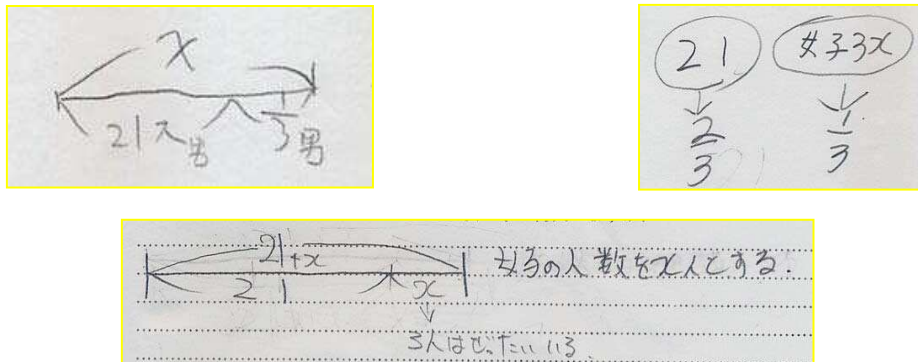Here are three examples from students' answers for item 6:

Figure 8. Samples of some ways students think of solving the equation.

For item 7, some students could not create the equation from the verbal statement, but they used a table structure to help them. One student who understood the meaning of the problem very well even though he could not create the equation, made this table for solving the problem.

Figure 9. Sample of how a student thinks of solving the equation.

Once again, when looking at the Japanese textbook one can easily understand why this happens. All of the examples in the textbook demonstrate how to understand the problem with the help of pictures, diagrams etc, as shown below:

Figure 10. Some examples from the Japanese textbook (pp. 86-87).

Thus the students learnt this idea to help them solve the verbal problems, and of course it is a very useful idea for students to use this way of analyzing, understanding and finally solving the problem as is already known (Polya, 1957).

Conclusion

Analysing the students' errors shows that the most frequently committed errors by both Thai and Japanese students were due to the mistakes in using the equalising property and failures in understanding the problem. However, Thai and Japanese students differ in the number and types of errors they made. The Thai students committed more types of errors

than the Japanese students, while the Japanese students were more aware of the processes of solving equations than the Thai students. Moreover the Japanese students showed that they have a tendency, although not high, to create different types of plans for solving the problems, by using diagrams and tables to analyze the problems and their structures.

Moreover, for all the problems, one of the causes for the way the students approach the problems is whether they have experienced problems like these before. For example with item 6, the pattern of the problem is from the textbook of the Thai secondary schools, which seems to have made it difficult for the Japanese students. On the other hand, with item 7, the pattern of the problem is from the Japanese textbook which might be the reason why it was so difficult for the Thai students to even attempt it.

In conclusion, there are several points which this study raises:

1. The same errors given by both Thai and Japanese student concerning the equalising property suggests that a good foundation or mastery of the previous learned concepts is essential for dealing with any equations.

2. Students are familiar with the particular process and the approach to solving equations that they have learned in class, and that they have read in their mathematics textbook. They have a tendency to use the familiar concepts and process to solve a problem by imitating the same process, without necessarily understanding the rationale behind it. This also inhibits their ability to handle non-routine problems, or problems for which they have not been taught the appropriate method.

3. In both countries the students' learning in their classes appears to be heavily dependent on their textbook, which then has a strong influence on their problem solving abilities. The teachers also use their textbook as a guide to teach the students, as is common in many countries. The Thai textbook emphasizes the process of solving problems but only has a few examples to demonstrate how to analyze problems and how to produce a plan to solve them. The Japanese textbook on the other hand shows many examples of how to analyze problems and how to produce plans for solving them by using diagrams, pictures, tables etc. Moreover the Japanese textbook gives tips under the example problems, which seem very helpful for students in finding the answers easily. Furthermore the Japanese textbook is more colorful and attractive to the students than the Thai textbook, which might make the ideas more appealing to the students.

4. The first author was familiar with the Thai schools, and was also able to visit several Japanese classrooms. She saw there that the Japanese teachers usually introduced the topic to the students by using "open-ended questions" on the board. They then let the students use manipulatives, aids, and other materials which the teachers had often prepared themselves for helping students to learn the concepts and processes involved. In fact it seemed that the Japanese students were familiar with solving problems by creating plans, from the kindergarten level through to their higher education. This process thus became part of the Japanese students' approach to solving problems in general. By comparison, the Thai teachers teach students by using mainly "chalk and talk", which means that the students have to follow the ideas which are given to them by the teacher. They therefore seem to lack the ability to create ideas to solve problems by themselves.

In conclusion, the study has shown that rather than just using mathematics tests in international comparisons to focus on total right answer scores, it might be much more productive to develop more cross-cultural studies of student errors and their possible causes. Researchers already know that one learns more from students' errors than from their right answers, and linking these errors to possible causes such as teachers' teaching

styles or, as in this case, textbook features such as problem structures, example problems, and problem-solving hints, will increase our knowledge enormously.

Acknowledgement

References

Bednarz, N., Kieran, C., & Lee, L. (Eds.). (1996). *Approaches to algebra; Perspective for research and teaching*. Dordrecht, the Netherlands: Kluwer Academics Publishers.

Booth, L. (1984). *Algebra: Children's strategies and errors*. Windsor, Bershire: NFER-Nelson Publishers Company Ltd.

Herscovics, N. (1989). Cognitive obstacles encountered in the learning of algebra. In S. Wagner & C. Kieran (Eds.), *Research issues in the learning and teaching of algebra* (pp. 60-86). Virginia, the United States of America: Lawrence Erlbaum Associates.

Kang, W., & Kilpatrick, J. (1992). Didactic transposition in mathematics textbooks. *For the Learning of Mathematics*, *12*(1), 2-7.

Kieran, C. (1992). The learning and teaching of school algebra. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 390-419). New York: McMillan Publisher Co.

Kittiwisit, S. (1994). *A construction of a mathematics diagnostical test focusing on equation*. Khon Kaen University, Khon Kaen, Thailand.

Love, E., & Pimm, D. (1996). 'This is so': A text on texts. In A. J. Bishop, K. Clements, C. Keitel, J. Kilpatrick & C. Laborde (Eds.), *International handbook of mathematics education* (Vol. 4, pp. 371-409). Dordrecht, the Netherlands: Kluwer Academics Publishers.

MONBUSHO. (1997). Mathematics textbook of first grade of lower secondary school in Japan. Japan; (Japanese Version).

Polya, G. (1957). *How to solve it*. Garden city, New York: Doubleday Anchor Books.

Appendix: Algebra Test

The algebra test consists of 4 simple equations and 3 equations problems as follow;

1. $2x - 5 = 17$
2. $6x - 3 = 2x - 19$
3. $5x - 3/2 = 1$
4. $5x + 3 = 3x - 5$

5. Sayoo-jan has some money in her wallet. Her father gave her three times more than she already has. She now has 1000 yen. How much money did she have before her father gave her any?
6. There are 21 boys in a class and the girls comprise 3 more than 1/3 of all the students. How many girls in a class?
7. At present Boy A is 12 years old and Mr. B is 39 years old. When will the age of Mr.B become twice the age of Boy A?